Localization of random copolymers

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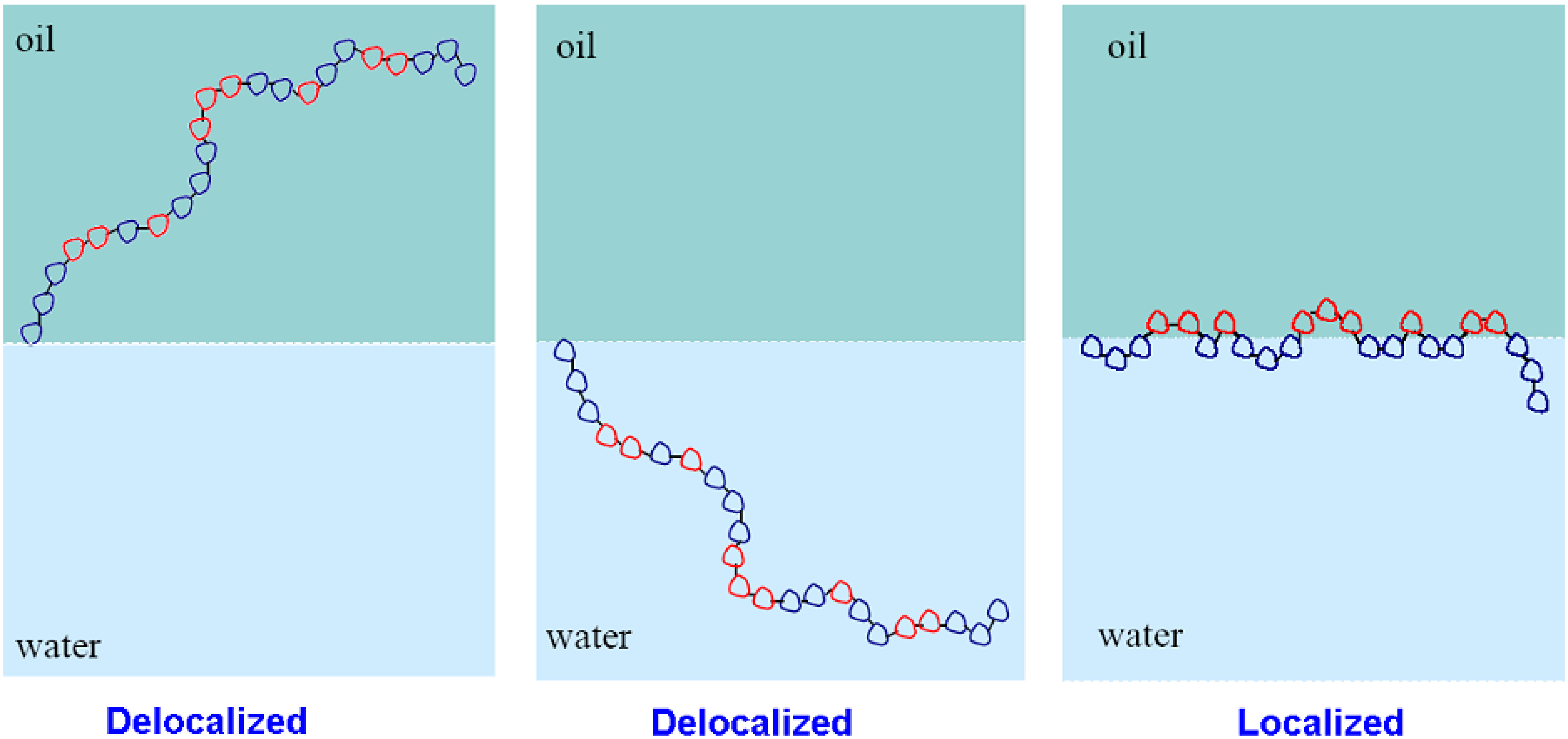
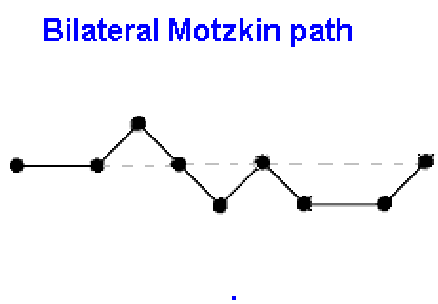
Introduction to localization of random copolymers

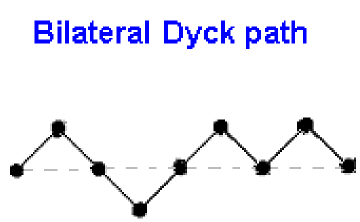


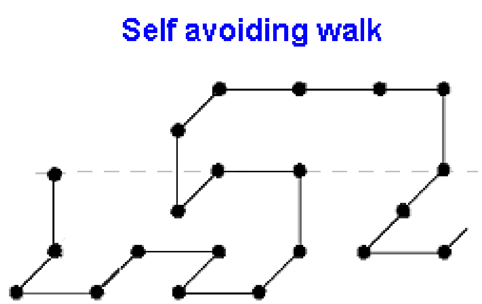
Polymer, Homopolymer, Copolymer, Random copolymer

Assumptions

- Dilute solution so polymer-polymer interactions can be ignored.
- System in equilibrium.
- Polymer's conformations: self-avoiding walks on a lattice, bilateral Motzkin paths, bilateral Dyck paths, etc.







- Two types of monomers: A and B.

$$Z_n(T|\chi) \;=\; \sum_{\omega} e^{-E(\omega|\chi)/kT}$$

- n: degree of polymerization (number of monomers)
- ω : conformation
- χ: sequence of monomer type (coloring)
  - Quenched randomness
- $E(\omega|\chi)$: energy of conformation ω for fixed χ.
- k: Boltzmann's constant.
- T: temperature
- Conformations (ω) with same energy are equally likely.

- In particular:

$$-E(\omega|\chi)/kT = \alpha v_A + \beta v_B + \gamma w$$

  - v_A is the number of type A vertices above the interface.
    - α is the contribution from each of these vertices to the energy.
  - v_B is the number of type B vertices below the interface.
    - β is the contribution from each of these vertices to the energy.
  - w is the number of vertices (A and B) at the interface.
    - γ is the contribution from each of these vertices to the energy.

$$Z_n(\alpha,\beta,\gamma|\chi) = \sum_{v_A,v_B,w} c_n(v_A,v_B,w|\chi)e^{\alpha v_A+\beta v_B+\gamma w}$$

  - $c_n(v_A,v_B,w|\chi)$ is the number of walks with v_A type A vertices above the interface, v_B type B vertices below the interface, and w vertices at the interface, given the coloring χ

- We are interested in the limiting quenched average free energy

$$\bar{\kappa}(\alpha, \beta, \gamma) = \lim_{n \to \infty} \left\langle \frac{\log Z_n(\alpha, \beta, \gamma | \chi)}{n} \right\rangle_\chi$$

  where $< \cdot >_\chi$ denotes expectation over χ.

- This will tell us the preferred phase of the system:
  - Delocalized into the water phase
  - Delocalized into the oil phase
  - Localized

- There are phase transitions
  - Point of non-analyticity of $\bar{\kappa}(\alpha, \beta, \gamma)$

- Hard problem

- Consider bounds
  - Limiting annealed average free energy

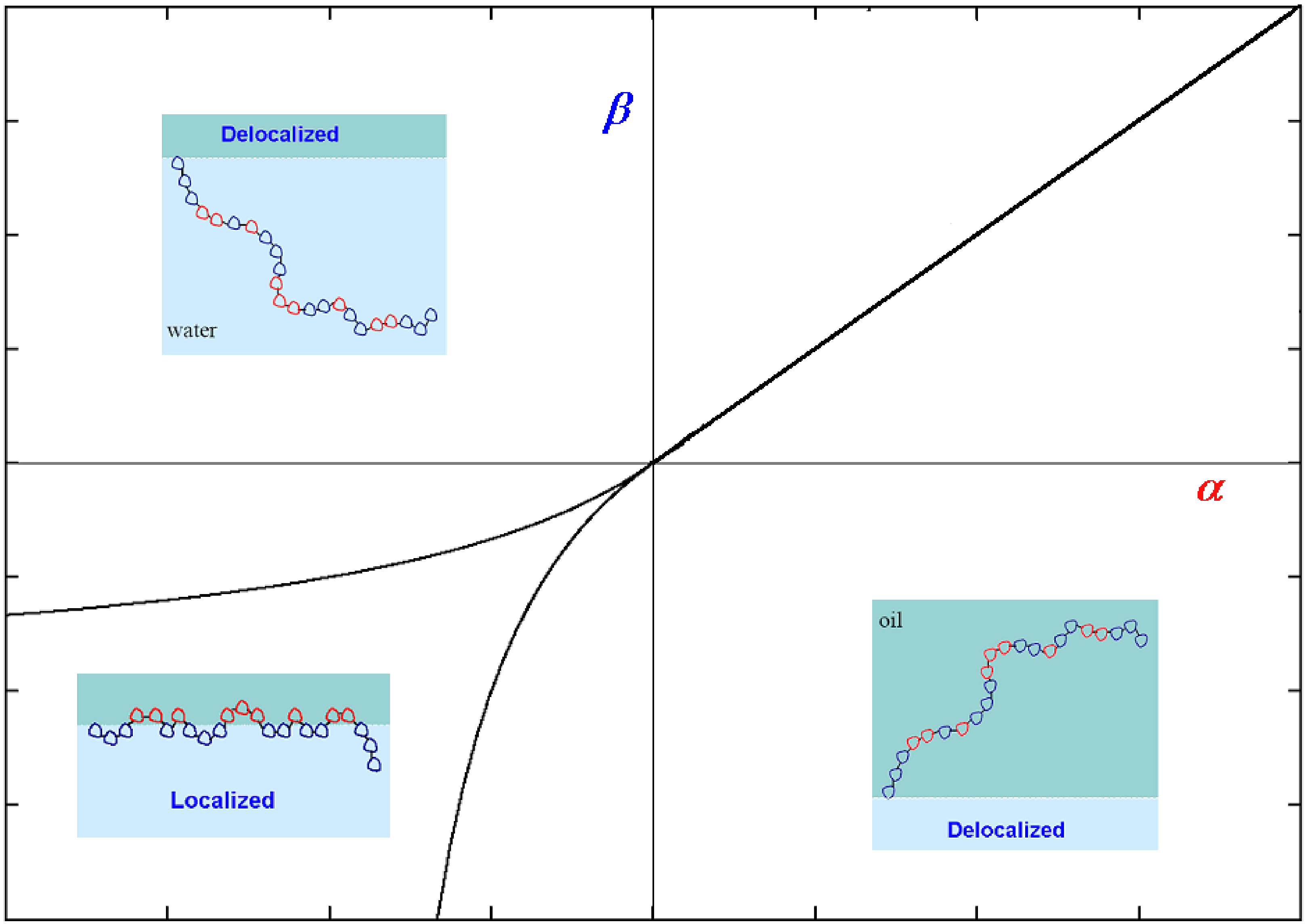
$$\bar{\kappa}^a(\alpha, \beta, \gamma) = \lim_{n\to\infty} \frac{\log \langle Z_n(\alpha, \beta, \gamma|\chi)\rangle}{n} \geq \bar{\kappa}(\alpha, \beta, \gamma)$$

  - Limiting constrained annealed average free energy (also known as the *Morita approximation* to the limiting quenched average free energy):

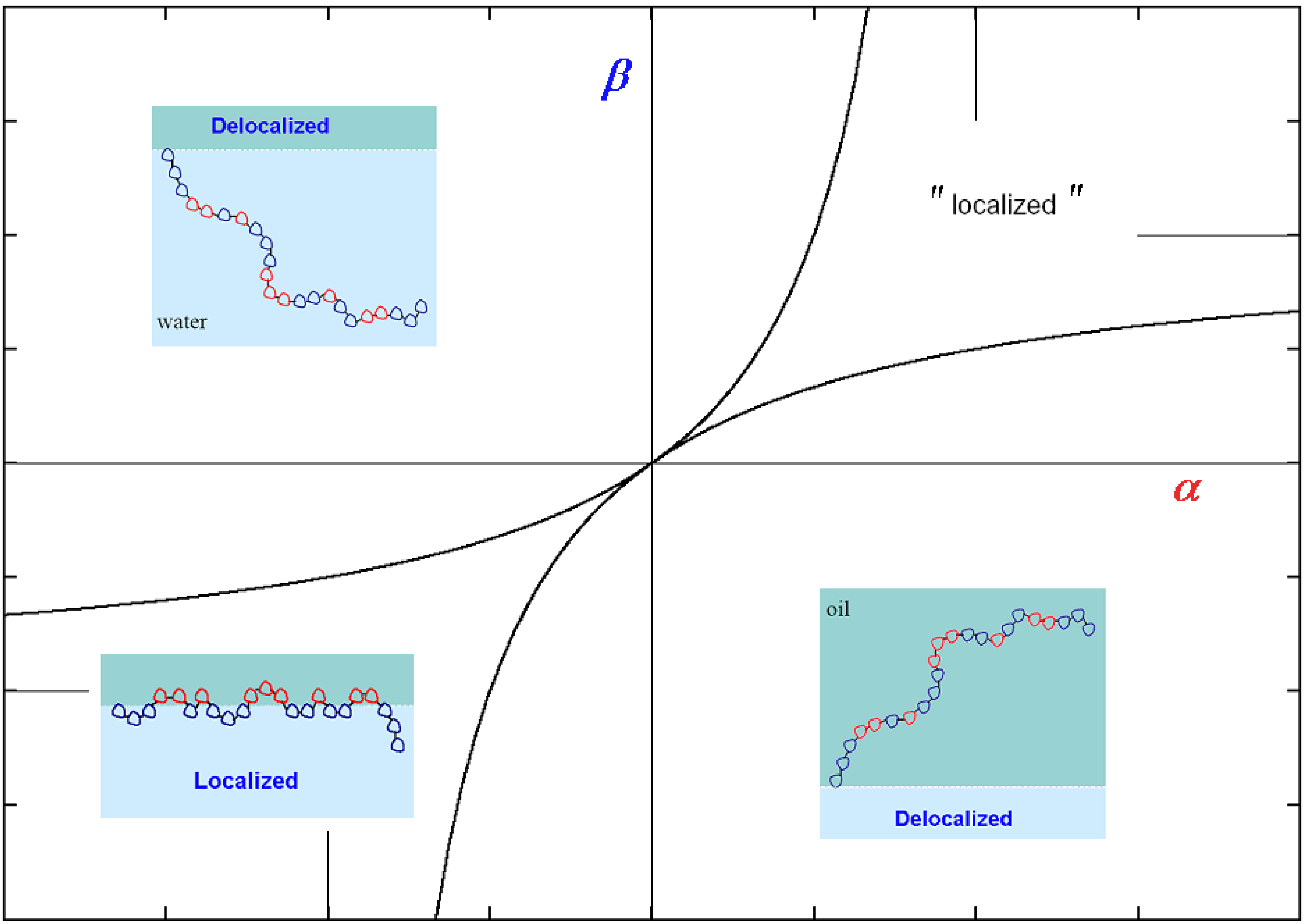
$$\bar{\kappa}^m(\alpha, \beta, \gamma) = \lim_{n\to\infty} \frac{\log \langle Z_n(\alpha, \beta, \gamma|\chi, \lambda_1, \lambda_2, \ldots)\rangle}{n} \geq \bar{\kappa}(\alpha, \beta, \gamma)$$

    where $\lambda_1, \lambda_2, \ldots$ are the Lagrange multipliers corresponding to constraints on the distribution of χ

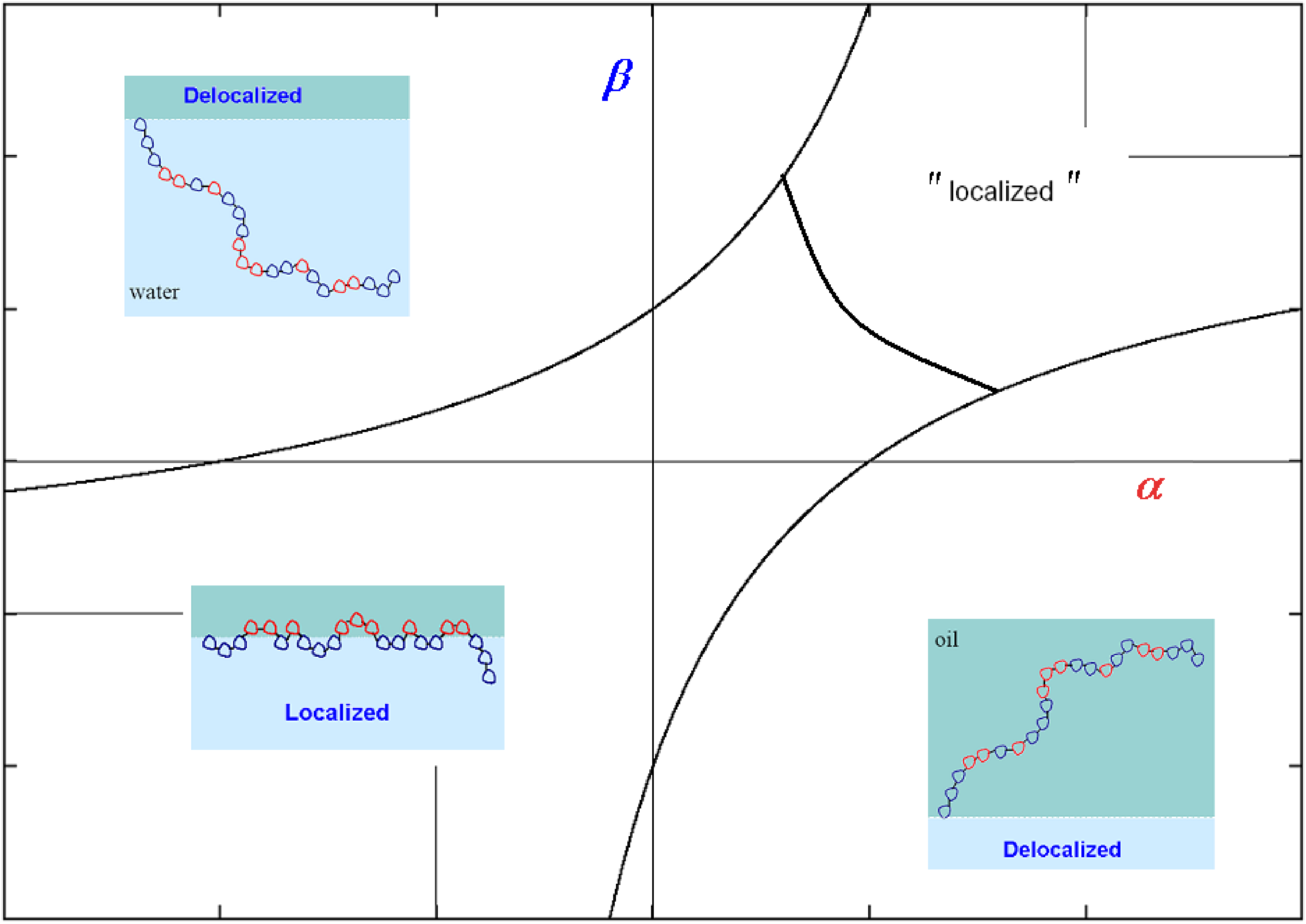
Annealed, $\gamma \leq 0$



Morita, $\gamma \leq 0$



Morita, $0 < \gamma_1^a < \gamma < \gamma_2^a$



Thanks.